

01

Continuity**Continuity of a Function at a Point**

Suppose f is a real function on a subset of the real numbers & let c be a point in the domain of f . Then f is continuous at c if

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Continuity of a Function in an Interval

Suppose f is a function defined on a closed interval $[a, b]$, & then f is continuous, it needs to be continuous at every point in $[a, b]$ including the end points a & b .

Continuity of f at a , $\lim_{x \rightarrow a^+} f(x) = f(a)$

Continuity of f at b , $\lim_{x \rightarrow b^-} f(x) = f(b)$

A function which is not continuous at point $x=c$ is said to be discontinuous at that point

02

Algebra of Continuous Functions

Theorem 1: Suppose f & g be two real functions continuous at a real number c , Then

- (1) $f + g$ is continuous at $x=c$ (3) $f \cdot g$ is continuous at $x=c$
 (2) $f - g$ is continuous at $x=c$ (4) f/g is continuous at $x=c$,
 (provided $g(c) \neq 0$)

Theorem 2: Suppose f & g are real valued functions such that $(f \circ g)$ is defined at c . If g is continuous at c & if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c .

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Implicit Functions

An equation of the form $f(x, y) = 0$ in which y is not expressible in terms of x is called an implicit function of x & y .

Derivative of Implicit Functions

Let $y=f(x, y)$, where $f(x, y)$ be an implicit function of x & y .
 Firstly differentiate both sides of equation w.r.t x

Then take all terms involving $\frac{dy}{dx}$ on L.H.S. & remaining terms on R.H.S. to get the required value.

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Differentiation of Inverse Trigonometric Functions

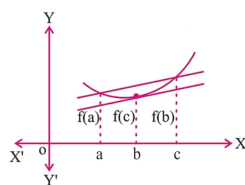
| $f(x)$ | $f'(x)$ | Domain of f' |
|------------------------------|------------------------------|----------------|
| $\sin^{-1}x$ | $\frac{1}{\sqrt{1-x^2}}$ | $(-1,1)$ |
| $\cos^{-1}x$ | $\frac{-1}{\sqrt{1-x^2}}$ | $(-1,1)$ |
| $\tan^{-1}x$ | $\frac{1}{1+x^2}$ | \mathbb{R} |
| $\cot^{-1}x$ | $\frac{-1}{1+x^2}$ | \mathbb{R} |
| $\sec^{-1}x$ | $\frac{1}{ x \sqrt{x^2-1}}$ | $ x > 1$ |
| $\operatorname{cosec}^{-1}x$ | $\frac{-1}{ x \sqrt{x^2-1}}$ | $ x > 1$ |

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Mean Value Theorem

If $f: [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$ & differentiable on (a, b) . Then there exists some c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



The Mean value Theorem states that there is a point c in (a, b) such that the slope of the tangent at $(c, f(c))$ is same as the slope of the secant between $(a, f(a))$ and $(b, f(b))$ or there is a point c in (a, b) such that the tangent at $(c, f(c))$ is parallel to the secant between $(a, f(a))$ & $(b, f(b))$

03

Differentiability

A function f is said to be differentiable at a point c in its domain, if its left hand & right hand derivatives exist at c are equal.

Here at $x = c$,

Left Hand Derivative,

$$L.H.D. = \lim_{h \rightarrow 0^-} \frac{f(c-h) - f(c)}{-h} = Lf'(c)$$

Right Hand Derivative,

$$R.H.D. = \lim_{h \rightarrow 0^+} \frac{f(c+h) - f(c)}{h} = Rf'(c)$$

Theorem: If a function f is differentiable at a point c , then it is also continuous at that point. Therefore, every differentiable function is continuous, but the converse is not true.

04

Algebra of Derivatives

Let u, v be the functions of X .

- (1) Sum and Difference Rule $(u \pm v)' = u' \pm v'$
 (2) Leibnitz or Product Rule $(uv)' = u'v + uv'$
 (3) Quotient Rule $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

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Chain Rule

If y is a function of u , u is a function of v & v is a function of x .

$$\text{Then, } \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$$

CONTINUITY AND DIFFERENTIABILITY

08

Logarithmic Differentiation

Logarithmic Differentiation is a very useful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$, where $f(x)$ & $u(x)$ are positive.

We apply logarithm (to base) on both sides to the above equation & then differentiate by using chain rule, in this way we can find $f'(x)$. This process is called logarithmic

$$\frac{d}{dx} (e^x) = e^x, \quad \frac{d}{dx} (\log x) = \frac{1}{x} \quad \& \quad \frac{d}{dx} a^x = a^x \log a$$

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Derivatives of Functions In Parametric Form

The set of equations $x = f(t)$, $y = g(t)$ is called the parametric form of an equation.

$$\text{Here, } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \text{ or } \frac{g'(t)}{f'(t)}$$

Here, $\frac{dy}{dx}$ is expressed in terms of parameter only without directly involving the main variables.

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Second Order Derivative

$$\text{Let } y = f(x), \text{ then } \frac{dy}{dx} = f'(x)$$

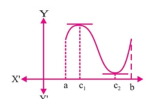
If $f'(x)$ is differentiable, then we may differentiate it again w.r.t. x & get the second order derivative represented by:

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) \text{ or } \frac{d^2y}{dx^2} \text{ or } f''(x) \text{ or } D^2y \text{ or } y'' \text{ or } y_2$$

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Rolle's Theorem

If $f: [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$ & differentiable on (a, b) such that $f(a) = f(b)$, then there exists some c in (a, b) such that $f'(c) = 0$



In the above graph, the slope of tangent to the curve at least at one point becomes zero. The slope of tangent at any point on the graph of $y = f(x)$ is nothing but the derivative of $f(x)$ at that point.